DELTA MATTER IN NEUTRON STARS IN A RELATIVISTIC QUARK MODEL

HIMANSHU SEKHAR SAHOO* RABINDRANATH MISHRA
RAVENSNAW UNIVERSITY, INDIA

PRAFULLA KUMAR PANDA
UTKAL UNIVERSITY, INDIA
INTRODUCTION

• Studies of the dense matter of neutron stars has assumed significance.

• Above the saturation density of nuclear matter the composition acquires mixture of uniform charge-neutral baryons and leptons.

• Hyperons and Delta baryons may appear in the inner core of neutron stars.

• This results in significant impact on the neutron star EoS.

• Leads to hyperon and $\Delta$ puzzle.
INTRODUCTION...

- Earlier studies indicated the formation of the $\Delta$ isobars at very high densities.
- We study the possibility of the early appearance of the $\Delta$ isobars at densities relevant to neutron stars.
- Constraining the meson - $\Delta$ coupling constants is a major problem.
- The effect of moderate variations in the meson - $\Delta$ coupling constants on the critical density of formation of $\Delta^-$ is studied.
The Relativistic Quark Model

- In such a picture the light quarks inside a bare nucleon are considered to be independently confined by a phenomenological average potential with an equally mixed scalar-vector harmonic form.

- The quarks in the baryon core are independently confined by an average flavor-independent potential $U(r)$ of the form:

$$U(r) = \frac{1}{2} (1 + \gamma^0) V(r)$$

$$V(r) = (ar^2 + V_0), \quad a > 0.$$
The Relativistic Quark Model

- The zeroth order quark dynamics inside the hadron-core is represented through the quark Lagrangian density as
  \[ \mathcal{L}^0_q(x) = \bar{\psi}_q(x) \left[ i\gamma^\mu \partial_\mu - m_q - U(r) \right] \psi_q(x) \]

- Appropriate corrections to mass of the hadron due to possible residual interactions:
  - Spurious center-of-mass motion of the ground state hadron.
  - Quark-Pion interaction arising out of the requirement for restoration of chiral symmetry at the SU(2)×SU(2) level.
  - Quark-Gluon interaction at short distances originating from one gluon exchange.

\[ M^*_B = E^0_B - \epsilon_{cm} + \delta M^\pi_B + (\Delta E^E_B)_g + (\Delta E^M_B)_g \]

Centre of mass correction.  Pionic correction  Gluonic corrections
The Equation of State

The Dirac Equation for individual quarks in the medium is:

$$\left[ \gamma^0 \left( \varepsilon_q - g_q^q \omega_0 - \frac{1}{2} g^q_\rho \tau_z \rho_{03} \right) - \vec{\gamma} \cdot \vec{p} - \left( m_q - g^q_\sigma \sigma_0 \right) - U(r) \right] \psi_q (\vec{r}) = 0$$

$$g^q_\sigma, g^q_\omega, g^q_\rho$$ Quark coupling constants with sigma, omega & rho mesons

In the Mean Field Approximation the meson fields are treated by their expectation values:

$$\sigma \rightarrow \langle \sigma \rangle \equiv \sigma_0 \quad \omega \rightarrow \langle \omega_\mu \rangle \equiv \delta_{\mu 0} \omega_0 \quad \rho \rightarrow \langle \rho_{\mu j} \rangle \equiv \delta_{\mu 0} \delta^{j3} \rho_{\mu j}$$

In the medium we have:

$$\varepsilon'_q = \left[ \varepsilon^*_q - \frac{V_0}{2} \right] \quad m'_q = \left[ m^*_q + \frac{V_0}{2} \right]$$

Where the effective quark energy & the effective quark mass are:

$$\varepsilon^*_q = \varepsilon_q - g_q^q \omega_0 - \frac{1}{2} g^q_\rho \tau_z \rho_{03} \quad m^*_q = m_q - g^q_\sigma \sigma_0$$

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The total energy density in the Mean Field Approximation is given as:

\[
\varepsilon = \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{\gamma}{(2\pi)^3} \sum_B \int_0^{k_f^B} d^3 k \sqrt{k_f^2 + M_B^*}^2 \\
+ \sum_l \frac{1}{\pi^2} \int_0^{k_f^l} k^2 dk \sqrt{k^2 + m_l^2}
\]

The Pressure in the Mean Field Approximation is given as:

\[
P = -\frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 \rho_{03}^2 + \frac{\gamma}{6\pi^2} \sum_B \int_0^{k_f^B} \frac{k^4 dk}{\sqrt{k_f^2 + M_B^*}^2} \\
+ \frac{1}{3} \sum_l \frac{1}{\pi^2} \int_0^{k_f^l} \frac{k^4 dk}{\sqrt{k^2 + m_l^2}}
\]
Constraints on the EoS

• Studies of nuclear matter at saturation density provide some constraints:

• Incompressibility (K): 242 MeV
  • Presently accepted range: 240±20 MeV ##

• Symmetry energy: J = 32 MeV

• Slope of the Symmetry energy: L=86.9 MeV
  • Presently accepted range: 58.7±28.1 MeV$$

Meson – Hyperon Coupling Constants

- The couplings of the hyperons to the $\sigma$ meson need not be fixed since we determine the effective mass of the hyperons self-consistently.

- The hyperon couplings to the $\omega$ meson are fixed by determining $x_{\omega B}$ fitting the hypernuclear potentials:

$$U_B = -(M_B - M_B^*) + x_{\omega B} g_\omega \omega_0$$

$$U_\Lambda = -28\,\text{MeV} \quad U_\Sigma = +30\,\text{MeV} \quad U_\Xi = -10\,\text{MeV}$$

- Isovector meson-hyperon coupling constants are fixed by:

$$g_{\rho Y} = g_{\rho N}$$

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Meson – Delta Coupling Constants

• The couplings of the $\Delta$ resonances are constrained poorly due to their unstable nature.

  – Studies based on quark counting argument: $x_{\omega\Delta} = 1, \ x_{\rho\Delta} = 1$
  – Wehberger et al., : $x_{\sigma\Delta} - x_{\omega\Delta} = 0.2$
  – Range of uncertainty of $\Delta$-potential by Drago et al., :
    
    $$-30 \text{MeV} + U_N \leq U_\Delta \leq U_N$$

  – Electron nucleus and Pion nucleus scattering:
    
    $$-90 \text{MeV} \leq U_\Delta \leq -50 \text{MeV}$$

  – Present work: $x_{\omega\Delta} = 0.7, \ x_{\rho\Delta} = 1, \ U_\Delta = -96 \text{MeV}$
  – Variations in the $g_{\omega\Delta}, \ g_{\rho\Delta}$ to study impact on $\rho_\Delta^{\text{crit}}$

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Particle fractions
Variation in $\rho-\Delta$ coupling strength

$m_q = 200$ MeV

$x_{\omega\Delta} = 0.7$
Effect of $x_{\omega\Delta}$ variation

<table>
<thead>
<tr>
<th>$x_{\omega\Delta}$</th>
<th>$M_{\text{max}}$ ($M_\odot$)</th>
<th>$R$ (km)</th>
<th>$R_{1.4}$ (km)</th>
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<tr>
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<td>1.86</td>
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<td>0.90</td>
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<td>2.11</td>
<td>11.89</td>
<td>13.6</td>
</tr>
</tbody>
</table>

PSR J0348+0432

$m_q=200$ MeV
$x_{\rho\Delta}=1.0$

$x_{\omega\Delta}=0.6$ - -
$x_{\omega\Delta}=0.7$
$x_{\omega\Delta}=0.8$
$x_{\omega\Delta}=0.9$
$x_{\omega\Delta}=1.0$
Effect of $x_{\rho\Delta}$ variation

<table>
<thead>
<tr>
<th>$x_{\rho\Delta}$</th>
<th>$M_{\text{max}}$ $(M_{\odot})$</th>
<th>$R$ (km)</th>
<th>$R_{1.4}$ (km)</th>
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<td>1.78</td>
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<td>1.85</td>
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<td>0.90</td>
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<td>11.93</td>
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<tr>
<td>1.00</td>
<td>1.98</td>
<td>12.08</td>
<td>13.6</td>
</tr>
</tbody>
</table>

PSR J0348+0432

$m_q=200$ MeV

$x_{\rho\Delta}=0.7$

$x_{\rho\Delta}=0.6$ ---

$x_{\rho\Delta}=0.7$ ---

$x_{\rho\Delta}=0.8$ ---

$x_{\rho\Delta}=0.9$ ---

$x_{\rho\Delta}=1.0$ ---
$\rho_{\Delta}^c$ vs. $M_\Delta$

Breit-Wigner Mass distribution

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Effect of variation in $\Delta$ mass

PSR J0348+0432

$m_q=200$ MeV

$M_\Delta=1110$ MeV
$M_\Delta=1232$ MeV
$M_\Delta=1350$ MeV
$m_q = 200 \text{ MeV}$

<table>
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<th>Composition</th>
<th>$M_{\text{max}}$ ($M_\odot$)</th>
<th>$R$ (km)</th>
<th>$\varepsilon_0$ (fm$^{-4}$)</th>
<th>$R_{1.4}$ (km)</th>
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<tbody>
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<tr>
<td>$N + \Delta$</td>
<td>1.98</td>
<td>12.08</td>
<td>5.56</td>
<td>13.6</td>
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<tr>
<td>$N + \Delta + \text{hyperons}$</td>
<td>1.90</td>
<td>12.41</td>
<td>5.02</td>
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CONCLUSIONS

• We study the possibility of early appearance of Δ isobars in neutron star matter.

• Critical density of formation is $2-3 \rho_0$.

• We observe a linear dependence of $\rho_{\Delta}^{\text{crit}}$ on the strength of vector coupling of the Δ, $x_{\rho\Delta}$.

• Appearance of Δ resonances shifts the formation of hyperons.

• The mass-radius relation of neutron stars depends on the values of $g_{\rho\Delta}$.

Our Collaborators

1. N Barik, Dept. of Physics, Utkal Univ.
2. T. Frederico, Instituto Tecnologico de Aeronatica, DCTA, Sao Jose dos Campos, SP, Brazil
3. Bao An Li, Texas A & M University, Commerce, Texas.

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