Different Clusterization Algorithms and Their Effect on Studying Cluster Correlations at Intermediate Energy Heavy-ion Collisions

Rohit Kumar
Department of Physics
Panjab University
Chandigarh, India
**Multifragmentation**

*Highly excited system formed during energetic nucleus-nucleus collisions breaks due to large excitation energy into Intermediate Mass Fragments [IMF’s], Light Charged Particles [LCPs] and Free Nucleons [FN].*

---

**Time Scale**

- **$t = 0$ fm/c**: Thermalization
- **$t_1$**: Thermalization
- **$t_2$**: Expansion (≈50 fm/c)
- **$t_3$**: Multi-Scission (≈100 fm/c)
Quantum Molecular Dynamics (QMD): Molecular Dynamics (MD) approach + n-body correlations + Equation of State (EOS) + Important Quantum Features (Pauli Principle, Stochastic Scattering, Particle Production)

**PRINCIPLE:**

**RUNS**

**INITIALIZATION**

**TIME**

**PROPAGATION**

**COLLISION**

The Nucleon - Nucleon Collisions:

Two nucleons can collide if they come closer than a distance of $\frac{\sigma_{\text{tot}}(\sqrt{s})}{\pi}$:

$\sigma_{\text{tot}}(\sqrt{s}) \rightarrow$ Total nn cross sections and $\sqrt{s} \rightarrow$ Centre of mass energy

- N + N $\rightarrow$ N + N
- N + N $\rightarrow$ N + Δ
- N + N $\rightarrow$ Δ + Δ
- N + Δ $\rightarrow$ N + N
- Δ $\rightarrow$ N + Δ
- N + Δ $\rightarrow$ N + Δ
- Δ + Δ $\rightarrow$ Δ + Δ

Pauli Blocking:

The collision is allowed with probability

$P_{\text{allowed}} = [1 - \min(P_1,1)][1 - \min(P_2,1)];$

$P_{\text{block}} = 1 - P_{\text{allowed}}$;
**Phase Space Generator**

**Quantum Molecular Dynamics (QMD):**
Molecular Dynamics (MD) approach + n-body correlations + Equation of State (EOS) + Important Quantum Features (Pauli Principle, Stochastic Scattering, Particle Production)

**PRINCIPLE:**

**RUNS**

**INITIALIZATION**

**TIME**

**PROPAGATION**

\[
\begin{align*}
\frac{d \mathbf{r}_i}{dt} &= \partial \frac{\langle H \rangle}{\partial \mathbf{p}_i} \\
\frac{d \mathbf{p}_i}{dt} &= -\partial \frac{\langle H \rangle}{\partial \mathbf{r}_i}
\end{align*}
\]

**COLLISION**

QMD model follows the time evolution of multi-nucleon phase space from the initial collision of projectile and target up to the final formation of fragments.

QMD will give us the phase space of the nucleons only, to identify the fragments, secondary algorithms are used.

**Secondary algorithms**

**How to identify fragments???: Big Problem**
Minimum Spanning Tree (MST) Method

- Two nucleons are bound if their distance in coordinate space fulfills:
  \[ |\mathbf{r}_i - \mathbf{r}_j| \leq d_{\text{min}} \quad d_{\text{min}} = 2 - 4 \text{fm} \]

- 30 years old

- Coordinate space correlations
- Neglected momentum space correlations
- Stability of fragments
- Only applicable to dilute matter
- No information of the compressed state

Problems??
Minimum Spanning Tree with momentum cut (MSTM) Method

In Minimum spanning tree method with momentum cut (MSTM) we impose additional constraint in momentum space.

- Two particles are bound if they fulfills following conditions:

  \[ | \mathbf{r}_i - \mathbf{r}_j | \leq 4\text{ fm} \]

  \[ | \mathbf{p}_i - \mathbf{p}_j | \leq P_{\text{fermi}} \]

Minimum Spanning Tree with energy cut (MSTE) method

- Checking the binding energies of pairs in clusters:

  \[ \mathcal{E}_{ij} \leq 0 \]

Stability check on MST fragments

- Two particles are bound if they fulfill binding energies criteria:

\[ s = \sum_{i=1}^{A_f} \left[ \frac{(\bar{p}_i - \bar{p}_{f,c})^2}{2m} + \sum_{i<j} V_{ij} \right] < E_{\text{Bind}} \]

- Binding energy of each fragment is calculated in the center of mass of the fragment.

- MST-B (1.1)-Binding energy of each fragment was compared with constant binding energy (-4 MeV/nucleon).

- MST-B (2.1)-Binding energy of each fragment is compared with the Modified Bethe-Weizsacker mass formula.

\[ E_{\text{MBW}} = a_v A_f - a_s A_f^{2/3} - \frac{a_c Z_f (Z_f - 1)}{A_f^{1/3}} - a_{\text{sym}} \frac{(A_f - 2Z_f)^2}{A_f(1 + \exp(- A_f / 17))} + \delta_{\text{new}} \]

Temperature-dependent binding energy cut (MSTBT)

Binding energy of each fragment is compared with the binding energies calculated using temperature-dependent binding energies

\[
\varsigma = \sum_{i=1}^{A_f} \left[ \frac{(\vec{p}_i - \vec{p}_f^{cm})^2}{2m} + \sum_{i<j}^{A_f} V_{ij} \right] < E^{\text{Thermal Bind}}
\]

\[
E^{\text{Thermal Bind}}(T) = \alpha(T)A_f + \beta(T)A_f^{2/3} + \left( \gamma(T) - \frac{\eta(T)}{A_f^{1/3}} \right) \left( \frac{4t_z^2 + 4|t_z|}{A_f} \right) \\
+ 0.8706 \frac{Z_f^2 R(0)}{A_f^{1/3} R(T)} \left( 1 - \frac{0.7636}{Z_f^{2/3}} - 2.29 \frac{R(0)^2}{[R(T)A_f^{1/3}]^2} \right) + \delta(T) \frac{f(A_f, Z_f)}{A_f^{3/4}}
\]

Comparison With Experimental Data

INDRA EXPERIMENTAL DATA

- Reactions - $^{129}$Xe+$^{119}$Sn, at 32 MeV/nucleon and $^{155}$Gd+$^{238}$U at 36 MeV/nucleon
- Central geometries
- Charge distribution as well as event-by-event fragment correlations are studied
- QMD coupled with MST, MSTM, MSTB, MSTBT calculations are shown

- Lesser number of IMFs are obtained
- Bigger size of largest fragment
- No QMD+ clusterization algorithm set reproduces data

J. D. Frankland et al., Nucl. Phys. A 689, 940 (2001)
**Comparison With Experimental Data**

**INDRA EXPERIMENTAL DATA**
- Reactions - $^{129}\text{Xe} + ^{119}\text{Sn}$, at 32 MeV/nucleon and $^{155}\text{Gd} + ^{238}\text{U}$ at 36 MeV/nucleon
- Central geometries
- Charge distribution as well as event-by-event fragment correlations are studied
- QMD coupled with MST, MSTM, MSTB, MSTBT calculations are shown

- No algorithm coupled with QMD reproduces data

Charge division between three largest fragments

J. D. Frankland et al., Nucl. Phys. A 689, 940 (2001)
Comparison With Experimental Data

INDRA EXPERIMENTAL DATA
• Reactions - $^{129}$Xe+$^{119}$Sn, at 32 MeV/nucleon and $^{155}$Gd+$^{238}$U at 36 MeV/nucleon
• Central geometries
• Charge distribution as well as event-by-event fragment correlations are studied
• QMD coupled with MST, MSTM, MSTB, MSTBT calculations are shown

• No algorithm coupled with QMD reproduces data

Need for new algorithm!!!!!

Multiplicity distribution of IMFs
J. D. Frankland et al., Nucl. Phys. A 689, 940 (2001)
Minimum Spanning Tree (MST) or its any extension cannot address the fundamental questions:

New approach: Assumptions
1. The nucleons from target and projectile are grouped in clusters and in free particles.
2. $N$-nucleons can form a fragment, if

$$\xi_i = \sum_{i=1}^{N^f} \left[ \sqrt{(p_i - P_{N^f}^{cm})^2 + m_i^2} - m_i + \frac{1}{2} \sum_{j \neq i}^{N^f} V_{ij}(x_i, x_j) \right] < E_{\text{Bind}} \times N^f,$$

where $E_{\text{Bind}} = -4.0$ MeV/nucleon.
3. Fragments don’t interact with each other.

---

Simulated Annealing

Simulated Annealing Clusterization Algorithm

\[ \zeta_c = \sum_{i=1}^{N_{Sl}} \left\{ \sqrt{(p_i - p_{cm}^{N_{Sl}})^2 + m_i^2} - m_i + \frac{1}{2} \sum_{j \neq i} V_{ij}(r_i, r_j) \right\} \]

\[ + \ldots \sum_{i=1}^{N_{Sp}} \left\{ \sqrt{(p_i - p_{cm}^{N_{Sp}})^2 + m_i^2} - m_i + \frac{1}{2} \sum_{j \neq i} V_{ij}(r_i, r_j) \right\} \]

\[ + \sum_{i=1}^{N_{Sm}} \left\{ \sqrt{(p_i - p_{cm}^{N_{Sm}})^2 + m_i^2} - m_i + \frac{1}{2} \sum_{j \neq i} V_{ij}(r_i, r_j) \right\} \]

\[ + \ldots \sum_{i=1}^{N_{Sn}} \left\{ \sqrt{(p_i - p_{cm}^{N_{Sn}})^2 + m_i^2} - m_i + \frac{1}{2} \sum_{j \neq i} V_{ij}(r_i, r_j) \right\} \]

\[ \Delta \zeta = \zeta_d - \zeta_c \]

-ve: accepted

+ve: exp [-\Delta E/c]  

\[ c: \text{initial temperature} \]

For Au+Au; 400\(^{400} = 16 \text{ million iterations} !! \]

R. K. Puri & Co-workers


Charge Distribution of Fragments

- QMD + SACA reproduces experimental data for all reactions
- Brownian One-Body (BOB) dynamics model + SIMON (de-excitation code) calculations also shown
- Stochastic-Initialization-Method (SIM) + SIMON (de-excitation code) calculations also shown
- Microcanonical Multifragmentation Model (MMM) with two fragment formation conditions
- Statistical Multifragmentation Model (SMM) with two source conditions

J. D. Frankland et al., Nucl. Phys. A 689, 940 (2001)
Higher Correlations among fragments

- QMD + SACA reproduces experimental data for all reactions
- Brownian-One-Body (BOB) dynamics model + SIMON (de-excitation code) calculations also shown
- Stochastic Initialization Method (SIM) + SIMON (de-excitation code) calculations also shown
- Microcanonical Multifragmentation Model (MMM) with two fragment formation conditions
- Statistical Multifragmentation Model (SMM) with two source conditions

J. D. Frankland et al., Nucl. Phys. A 689, 940 (2001)

SACA and Other model Calculations


IMF’s Multiplicity

The SACA algorithm is found to explain the experimental results consistently for all entrance channels.

The SACA method can lead us to better understand the reaction dynamics as fragments can be realized much earlier in time.

The spatial correlations are not enough to produce reliable results at lower incident energies.

This algorithm is also found to reproduce various parameters related to Phase-transition.

We are now extending this algorithm to study isospin effects in heavy-ion collisions.
All calculations shown here are done in collaboration with Professor Rajeev K. Puri, Panjab University, Chandigarh (INDIA)