Excitation modes and rotational moment of inertia in triaxial nuclei

Kouhei Washiyama
Research Center for Superheavy Elements, Kyushu Univ., Japan

Takashi Nakatsukasa
Center for Computational Sciences, Univ. Tsukuba, Japan

Introduction: Quadrupole shape fluctuations in transitional nuclei

Goal: 5 Dim. quadrupole collective Hamiltonian

Method: 3D QRPA with Skyrme energy density functional

Result: Strength functions of triaxial superfluid nuclei

Result: Moment of inertia by local FAM-QRPA

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Shape fluctuation

Quadrupole Shape fluctuation in **transitional region**

**106 Pd**

Potential is flat ($V < 2$ MeV) at large region

→ **Shape fluctuation**

Two minima at oblate and prolate region

→ **Shape coexistence**

**68 Se**

Go beyond mean field to describe shape fluctuation

Hinohara et al., PRC82 (2010) 064313
5D quadrupole collective (Bohr) model

Bohr Hamiltonian

\[ \mathcal{H} = T_{\text{vib}} + T_{\text{rot}} + V(\beta, \gamma) \]

\[ T_{\text{vib}} = \frac{1}{2} D_{\beta\beta}(\beta, \gamma) \dot{\beta}^2 + D_{\beta\gamma}(\beta, \gamma) \dot{\beta} \dot{\gamma} + \frac{1}{2} D_{\gamma\gamma}(\beta, \gamma) \dot{\gamma}^2 \]

\[ T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^{3} J_k(\beta, \gamma) \omega_k^2 \]

Quantize Hamiltonian

\[ \hat{H}\psi_{\alpha IM}(\beta, \gamma, \Omega) = E_{\alpha I}\psi_{\alpha IM}(\beta, \gamma, \Omega) \]

Excitation spectra
5D quadrupole collective (Bohr) model

HFB (V) + **Local QRPA (D_ββ', D_βγ', D_γγ', J_k)** with P+Q force


HFB (V) + **Cranking approx. mass (D_ββ', D_βγ', D_γγ', J_k)**

with modern Energy Density Functional (Skyrme, Gogny, Relativistic)

Prochniak et al., NPA730 (2004) 59
Niksic et al., PRC79 (2009) 034303
Delaroche et al., PRC81 (2010) 014303, etc.

**Goal**: Combine the two approaches

HFB (V) + **Local QRPA (D, J)** with **Skyrme EDF**

\[ V(\beta, \gamma) \] \( \beta, \gamma \)-constrained Skyrme HFB

\[ D_{\mu\nu}(\beta, \gamma) \] Local Skyrme QRPA on each \( \beta, \gamma \)
Aim of this talk

• To construct 5D quadrupole collective Hamiltonian with Skyrme EDF
• Skyrme QRPA for triaxial shapes is NOT available

**Step 1:** Construct Skyrme QRPA for triaxial shapes

Finite amplitude method (FAM)

FAM: method for efficiently solving QRPA

**Step 2:** Local FAM+QRPA at each $\beta$, $\gamma$

⇒ Collective mass

$$D_{\mu\nu}(\beta, \gamma), \quad \mathcal{J}_k(\beta, \gamma)$$
Finite amplitude Method (FAM)

Linear response TDDFT

\[
(E_{\mu} + E_\nu - \omega)X_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) = -F_{\mu\nu}^{20}
\]

\[
(E_{\mu} + E_\nu + \omega)Y_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{02}(\omega) = -F_{\mu\nu}^{02}
\]

\[
\delta H_{\mu\nu} = \frac{\partial H_{\mu\nu}}{\partial \mathcal{R}_{\alpha\beta}} \partial \mathcal{R}_{\alpha\beta}
\]

\[
\delta H_{\mu\nu}(\omega) = \frac{1}{\eta} \left\{ H_{\mu\nu}[\mathcal{R}_0 + \eta \delta \mathcal{R}(\omega)] - H_{\mu\nu}[\mathcal{R}_0] \right\}
\]

\(\mathcal{R}_0\): Ground state density

\(\delta \mathcal{R}\): Fluctuating density

\(\eta\): Small parameter

- Residual part → finite difference
- No need of QRPA \((A,B)\) matrices

QRPA: \(N^2 \times N^2\) matrix

FAM: \(N \times N\) matrix

- FAM code from HF\((B)\) code
- Equivalent to (Q)RPA

Nakatsukasa et al., PRC76 (2007) 024318
Avogadro & Nakatsukasa, PRC84 (2011) 014314
Stoitsov et al., PRC84 (2011) 041305
Liang et al., PRC87 (2013) 054310
Niksic et al., PRC88 (2013) 044327
Pei et al., PRC90 (2014) 051304
Mustonen et al., PRC93 (2016) 014304
Calculation set-up of FAM

• Based on 3D Cartesian coordinate HFB (cr8 code)

• FAM equation is iteratively solved at each $\omega$

  \[ X, Y \rightarrow \delta R \rightarrow \delta H^{20,02} \rightarrow \text{new } X, Y \ (\text{at fixed } \omega) \]

• $\omega \rightarrow \omega + i \gamma \quad \gamma = 0.5 \ \text{MeV}$

• Strength function

  \[ S(\omega) = -\frac{1}{\pi} \text{Im} \left( \sum_{\mu < \nu} F_{\mu \nu}^{20*} X_{\mu \nu}(\omega) + F_{\mu \nu}^{02*} Y_{\mu \nu}(\omega) \right) \]
Triaxially deformed superfluid nucleus

Isoscalar quadrupole response

\[ Q_{2K}^{(\pm)} \propto r^2 (Y_{2+K} \pm Y_{2-K}) \]

- **Five** different strength functions
- **Three spurious rotations** (x, y, z)
- 98-99% of energy-weighted sum for \( \omega < 50 \text{MeV} \) is satisfied

**Computation:**
15 min/\( \omega \) (16 threads)
3.5 GB memory

**Numerical set up**

\( ^{110}\text{Ru}: 17^3 \text{ mesh}, R_{\text{max}}=14.0\text{fm}, 1120 \text{ HF states} \)

\( ^{190}\text{Pt}: 19^3 \text{ mesh}, R_{\text{max}}=15.6\text{fm}, 1360 \text{ HF states} \)

KW, Nakatsukasa, PRC96, 041304(R) (2017)
Aim of this talk

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- Skyrme QRPA for triaxial shapes is NOT available

**Step 1:** Construct Skyrme QRPA for triaxial shapes

Finite amplitude method (FAM)

FAM: method for efficiently solving QRPA

**Step 2:** Local FAM+QRPA at each $\beta$, $\gamma$

$\Rightarrow$ Collective mass

$$T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^{3} J_k(\beta, \gamma) \omega_k^2$$
Moment of inertia from FAM

\[ T_{\text{rot}} = \frac{1}{2} \sum_{k=1}^{3} J_k(\beta, \gamma) \omega_k^2 \]

Spurious modes of QRPA ↔ Nambu-Goldstone (NG) modes

Mean field breaks symmetries (translational, rotational etc.)

Relation between Thouless-Valatin inertia \((M_{\text{NG}})\) and FAM

\[ S_{\text{FAM}}^\mu(\hat{P}_{\text{NG}}, \omega = 0) = \sum_{\mu < \nu} P_{\mu\nu}^{20*} X_{\mu\nu}(0) + P_{\mu\nu}^{02*} Y_{\mu\nu}(0) = -M_{\text{NG}} \]

\[ \hat{P}_{\text{NG}} = J_k, \quad M_{\text{NG}} = J_k^{\text{TV}} \]

for rotational moment of inertia

Hinohara, PRC92(2015)034321

Strength at only \(\omega = 0\) ↔ Small computations

( a few minutes with 16 threads for one \(\beta-\gamma\) point)
Result: Moment of inertia on $\beta, \gamma$ plane

$\mathcal{J}_{1}^{TV}(\beta, \gamma)$

$\mathcal{J}_{2}^{TV}(\beta, \gamma)$

$\mathcal{J}_{3}^{TV}(\beta, \gamma)$

Potential

$V(\beta, \gamma)$

At $\beta-\gamma$ 84 points (white dots), constrained HFB+ local FAM-QRPA with interpolation in between

Numerical set up

17$^3$ mesh, $R_{\text{max}}=14.0\text{fm}$, 1120 HF states, volume pairing
Importance of residual effect

Thouless-Valatin moment of inertia ← With residual interaction

vs.

Inglis-Belyaev moment of inertia ← Without residual interaction

(Cranking mass)

- Moment of inertia is increased by the residual interaction
  \( \mathcal{J}^{TV} / \mathcal{J}^{IB} > 1 \)
- \( \beta - \gamma \) dependence is important
- Use of \( J^{IB} \) or constant factor on \( J^{IB} \) is not enough

KW, Nakatsukasa, arXiv:1803.06828
Summary

Shape fluctuation → Large amplitude collective motion

3D FAM with Skyrme EDF for triaxial superfluid nuclei

Moment of inertia by Local FAM+QRPA

Future plan

Local FAM+QRPA \( D_{\beta\beta}, D_{\beta\gamma}, D_{\gamma\gamma} \)

5D quadrupole collective (Bohr) Hamiltonian